A Universe Dominated by Dilaton Field

Chang Jun Gao[1*] and Shuang Nan Zhang[1,2,3,4†]

[1]*Department of Physics and Center for Astrophysics,*
Tsinghua University, Beijing 100084, China(mailaddress)
[2]*Physics Department, University of Alabama in Huntsville, AL 35899, USA*
[3]*Space Science Laboratory, NASA Marshall Space Flight Center, SD50, Huntsville, AL 35812, USA and*
[4]*Key Laboratory of Particle Astrophysics, Institute of High Energy Physics,*
Chinese Academy of Sciences, Beijing 100039, China

Using a single dilaton field, a unified model of the Universe is proposed, which evolves from the radiation-like dominance in the Big Bang, to the dark-matter-like dominance in the early Universe, to the coexistence of both dark-matter-like and dark energy today, and finally to the dark energy dominance in the infinite future. This model is consistent with current results on the age of the Universe, the transition redshift from deceleration to acceleration, BBN and evolution of dark energy. Future higher quality data may constrain the cosmic evolution of dark matter, dark energy and Hubble constant more precisely and make critical tests on our model predictions.

PACS numbers: 98.80.Cq, 98.65.Dx

I. INTRODUCTION

The observations of supernova [1, 2], cosmic microwave background [3, 4] and large scale structure [5, 6] in recent years indicate that the Universe is accelerating and thus some form of dark energy must exist in the Universe to drive this acceleration. Investigation on the nature of dark energy becomes one of the most important tasks for modern physics and modern astrophysics. Up to now, many candidates of dark energy have been proposed to fit various observations which include the Einstein's cosmological constant [7], quintessence [8], k-essence [9], tachyon [10], phantoms [11], brans [12] and so on. There have also been some models of unified dark matter and dark energy [13]. Padmanabhan and Choudhury developed a model of unified matter and dark energy using a single tachyon field to explain both clustered dark matter at small scales and smooth dark energy at large scales. Gonzalez-Diaz investigated a model of the Universe filled with a generalized Chaplygin fluid which mimics both dark matter and dark energy. Cardone et al have used a phenomenological approach to study the problem of unified dark matter and dark energy. Susperregi has also developed a cosmological scenario where the dark matter and dark energy are two simultaneous manifestations of an inhomogeneous dilaton field. In his study, Susperregi has constructed a dilaton potential with the "trough" feature.

In this paper, we present a model of the Universe dominated by the dilaton field with a Liouville type potential. The potential is the counterpart of the Einstein's cosmological constant in the dilaton gravity theory. Since it can be reduced to the Einstein cosmological constant when the dilaton field is set to zero,

we call it the cosmological constant term in the dilaton gravity theory.

II. DYNAMIC EQUATIONS

In Ref.[14], we have derived the action of dilaton field in the presence of Einstein's cosmological constant

$$S = \int d^4x \sqrt{-g} \left[R - 2\partial_\mu \phi \partial^\mu \phi - V(\phi) \right], \qquad (1)$$

where

$$
\begin{aligned}
V(\phi) &= \frac{2\lambda}{3(1+\alpha^2)^2} \left[\alpha^2 \left(3\alpha^2 - 1\right) e^{-2\phi/\alpha} \right. \\
&\left. + \left(3 - \alpha^2\right) e^{2\phi\alpha} + 8\alpha^2 e^{\phi\alpha - \phi/\alpha} \right], \qquad (2)
\end{aligned}
$$

is the Liouville-type potential with respect to the cosmological constant, ϕ is the dilaton field, λ is the Einstein's cosmological constant and α is a free parameter which governs the strength of the coupling of the dilaton to the Einstein's cosmological constant. When $\alpha = 0$ or $\phi = 0$, the action reduces to the usual Einstein scalar theory and the potential becomes a pure cosmological constant. Here the usual Einstein's cosmological constant term appears not as a constant but as a coupling to the dilaton field which reveals the interaction of vacuum energy (i.e. the dark energy today) and dilaton matter. This implies that the potential plays the role of both matter and dark energy. In other words, matter and dark energy might both originate from this unique potential. This is the starting point of our discussion. In the following we will investigate whether this potential can mimic the total energy of our Universe. When $\alpha = \pm\sqrt{1/3}, \pm 1, \pm\sqrt{3}$, the action is just the SUSY potential in sting theory. It is apparent that changing the sign of α is equivalent to changing the sign of ϕ. Thus it is sufficient to consider

only $\alpha > 0$ while ϕ may be positive or negative.

Recent measurements of the power spectrum of the cosmic microwave background detected a sharp peak around $l \approx 200$, indicating that the Universe is highly likely flat. Thus we only consider the flat Universe which is dominated by the spatially homogeneous dilaton field and described by the flat Friedmann-Robertson-Walker metric.

The equations of motion can be reduced to three equations

$$\frac{\dot{a}^2}{a^2} + \frac{2\ddot{a}}{a} = -\dot{\phi}^2 + \frac{1}{2}V, \quad 3\frac{\dot{a}^2}{a^2} = \dot{\phi}^2 + \frac{1}{2}V,$$
$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{1}{4}\frac{\partial V}{\partial \phi}, \tag{3}$$

where dot denotes the derivative with respect to t and $a(t)$ is the scale factor of the Universe. We find that the last equation can be derived from the former two. So it is sufficient to solve the first and the second equation. It follows immediately that

$$3H^2 = \left(\frac{dH}{d\phi}\right)^2 + \frac{1}{2}V, \tag{4}$$

where $H[\phi(t)] \equiv \dot{a}/a$ is the Hubble parameter.

For an arbitrary α, a solution of Eq.(4) is given by

$$H = \frac{1}{1+\alpha^2}\sqrt{\frac{\lambda}{3}}\left(e^{\phi\alpha} + \alpha^2 e^{-\phi/\alpha}\right). \tag{5}$$

Since there is no integration constant in the solution $H(\phi)$, it is apparent $H(\phi)$ is a special solution to Eq.(4). We are interested in that this special solution can actually describe the evolution of our Universe. After inserting $H(\phi)$ into Eqs.(3), we can obtain the expressions of $a(\phi)$ and $\phi(t)$. So the equations of motion are resolved.

III. MIMIC OUR UNIVERSE

We can now mimic the total energy of the Universe using the single dilaton field. Taking into account of $\alpha > 0$, we find from $dH/d\phi = -\dot{\phi}$ that when $\phi > 0$, ϕ is the monotonically decreasing function of $t \in (-\infty, +\infty)$. When $\phi < 0$, ϕ is the monotonically increasing function of $t \in (-\infty, +\infty)$. Thus the positive dilaton field can never evolve to the negative one and vice versa. Without the loss of generality, we consider only the case of $\phi > 0$.

From the equations of motion we can derive the scale factor of the Universe $a(\phi)$, the energy density of dilaton field ρ, the age of the present Universe τ and the parameter of equation of state $w = p/\rho$,

$$a = e^{-\int_{\phi_0}^{\phi} H(dH/d\phi)^{-1}d\phi}, \quad \rho = \frac{3}{8\pi}H^2,$$
$$\tau = -\int_{\phi_i}^{\phi_0}\left(\frac{dH}{d\phi}\right)^{-1}d\phi, \quad w = 1 - \frac{V}{8\pi\rho}, \tag{6}$$

where ϕ_i is the value of the field at the time of $t = t_p$ (t_p is the Planck time) and ϕ_0 is the value today. Since the evolution of the Universe is from ϕ_i to ϕ_0, so ϕ_i is the lower limit and ϕ_0 is the upper one. We have set the scale factor today equals to 1.

Among the equations of motion only two are independent, we can now recall the specific initial conditions with respect to $\rho(\phi)$ and $w(\phi)$ in Eqs.(6). Throughout the paper we will employ the Planck units in which the speed of light c, gravitational constant G and Planck constant $\hbar$ are all set equal to 1. Then we have the Planck energy density $\rho_p = 1$, Planck time $t_p = 1$. Since our discussion is starting from the Planck time when the Universe was filled with the extremely relativistic particles (equation of state $p = 1/3\rho$) as required by the standard Big Bang model, so we have the energy density $\rho_i = 1$ and state equation $w_i = 1/3$ at the time of $t = t_p$. Given any value of α in the expression of w, we can numerically calculate the relationship between the equation of state w and ϕ, as shown in Fig.1. We find that when $\alpha = 1.414213562 \simeq \sqrt{2}$ and $\phi \to \infty \Longleftrightarrow z \to \infty$, $w \to 1/3$, which corresponds to the equation of state of the Universe at the Planck time. Consequently we take $\alpha = \sqrt{2}$ in the rest of the paper.

For the present universe, it has the critical energy density (to make it flat) and that it consists of $\Omega_{m0} = 0.30 \pm 0.04$ [15] of matter and $\Omega_{x0} = 0.70 \pm 0.04$ [15] of dark energy. Then from the current parameters of equation of state, for matter $w_{m0} = 0$ and dark energy $w_{x0} = -1.02^{+0.13}_{-0.19}$ [16], we get the parameter of the equation of state for the total energy $w_0 = w_{x0}\Omega_{x0}/(\Omega_{m0} + \Omega_{x0})$; the critical energy density $\rho_0 = 3.6h^2 \times 10^{-123}$, where h is in the range of $0.70^{+0.04}_{-0.03}$ [15] for the present Universe. In Table I, we summarize the age of the Universe τ and the transition redshift from deceleration to acceleration z_T evaluated with different combinations of Ω_{m0}, w_{x0} and h within their ranges given above. For brevity, we only show these certain combinations whose predictions are consistent with current results on the age of the Universe and the transition redshift.

Now let's inspect carefully whether the model satisfies the constraints from the astronomical observations and the standard Big Bang model.

1.Age of the Universe

Table I shows that many combinations predict the ages of the Universe in agreement with $12.6^{+3.4}_{-2.4}$ Gyr determined from globular clusters age [17] and 12.5 ± 3.5 Gyr from radioisotopes studies [18]. It also doe not conflict the result of $14.1^{+1.0}_{-0.9}$ Gyr from WMAP, ADSS and SN Ia data [15].

2.Transition Redshift of the Universe

The range of transition redshift from deceleration to acceleration of the Universe is constrained by ΛCDM [19] as $z_T = 0.67$, the current most stringent constraints of combined GRB+SN Ia data [20] $z_T = 0.73 \pm 0.09$ and the joint analysis of SNe+CMB data $z_T = 0.52 \sim 0.73$ [21]. To satisfy the constraint of $z_T = 0.52 \sim 0.82$, it

TABLE I: Cosmic parameters evaluated with different Ω_{m0}, w_{x0} and h. The dimensions: $\tau(10^{10}yr)$.

Initial			Conditions ‖	Results	
Ω_{m0}	w_{x0}	h	‖	τ	z_T
0.26	-0.92	0.67	‖	1.32	0.52
0.26	-0.92	0.70	‖	1.26	0.52
0.26	-0.92	0.74	‖	1.19	0.52
0.26	-1.00	0.67	‖	1.41	0.68
0.26	-1.00	0.70	‖	1.35	0.68
0.26	-1.00	0.74	‖	1.28	0.68
0.26	-1.055	0.67	‖	1.49	0.82
0.26	-1.055	0.70	‖	1.43	0.82
0.26	-1.055	0.74	‖	1.35	0.82
0.30	-0.97	0.67	‖	1.31	0.52
0.30	-0.97	0.70	‖	1.26	0.52
0.30	-0.97	0.74	‖	1.19	0.52
0.30	-1.00	0.67	‖	1.34	0.57
0.30	-1.00	0.70	‖	1.28	0.57
0.30	-1.00	0.74	‖	1.22	0.57
0.30	-1.115	0.67	‖	1.49	0.82
0.30	-1.115	0.70	‖	1.43	0.82
0.30	-1.115	0.74	‖	1.35	0.82
0.34	-1.03	0.67	‖	1.31	0.52
0.34	-1.03	0.70	‖	1.26	0.52
0.34	-1.03	0.74	‖	1.19	0.52
0.34	-1.185	0.67	‖	1.49	0.82
0.34	-1.185	0.70	‖	1.43	0.82
0.34	-1.185	0.74	‖	1.35	0.82

FIG. 1: $w - \phi$ relations. ($\phi \to \infty \Longleftrightarrow z \to \infty$). Dotted curve is for $\alpha = \sqrt{2}$. When $\alpha = \sqrt{2}$ and $\phi \to \infty$, we have $w = 1/3$ which is for the radiation-like dominated Universe. For any α, $w \to -1$ when $\phi \to 0 \Longleftrightarrow z \to -1$.

FIG. 2: Evolutions of the dark energy for $w_{x0} = -1.058$, overplotted on the 1-σ and 2-σ exclusion zones obtained by combining supernova Ia and WMAP observations [Fig.(2) (upper panel) in Jassal et al].

is required that w_{x0}, i.e., $w_{x0} = -1.055 \sim -0.92$ for $\Omega_{m0} = 0.26$, $w_{x0} = -1.115 \sim -0.97$ for $\Omega_{m0} = 0.30$ and $w_{x0} = -1.185 \sim -1.03$ for $\Omega_{m0} = 0.34$.

3.Big Bang Nucleosynthesis

The energy density of the early Universe can be approximated as $\rho = g_*\pi^2 T^4/30$, where T denotes the temperature and g_* denotes the effective number of degrees of freedom by taking into account the variety of particles at higher temperatures. We have $g_* = 10.75$ and $g_* = 3.36$ at the temperatures $T = 10$ MeV and $T = 0.1$ MeV, respectively [22]. The energy density required by BBN epoch is between 1.57×10^{-84} ($T = 10$ MeV) and 4.92×10^{-93} ($T = 0.1$ MeV). Then we have approximately the beginning time of BBN $\tau_{bbn1} \simeq 7.45$ msec and the ending time of BBN $\tau_{bbn2} \simeq 133$ sec. So the model doe not conflict the result of $10^{-2} \sim 10^2$ sec estimated by the BBN theory [22].

4. Dark Energy

When the dilaton field ϕ approaches zero, i.e. the cosmic time t approaches infinity, we have $\rho = -p = \lambda/(8\pi)$. Then the Universe evolves to de Sitter Universe in the distant future. So whether the dark energy of the present Universe is quintessence $w_x > -1$ or phantom $w_x < -1$, it will evolve into the pure cosmological constant in the future. Frankly, we can not distinguish the dark energy component from the total energy density in general. However, since the matter scales as $\rho_m = \rho_{m0}/a^3$ for low redshift, we can approximate the dark energy as
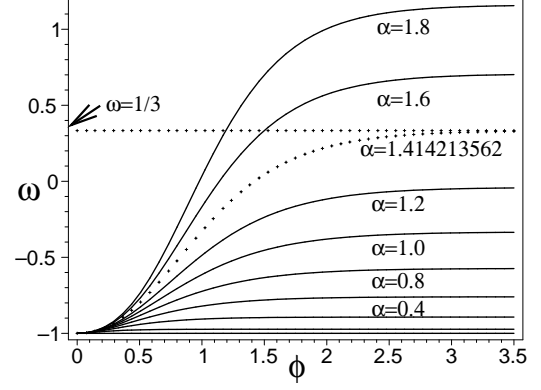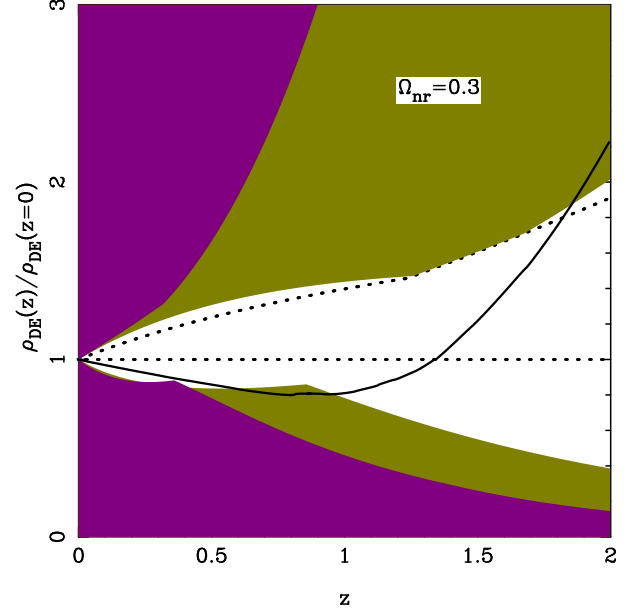
$\rho_x = \rho - \rho_m$. Because the dark energy was less important in the past, the most sensitive redshift interval for probing dark energy is $z = 0 \sim 2$. Current observations of SN Ia, CMB and LSS have made constraints [23] on the evolution of ρ_x at $z = 0 \sim 2$. FIG.2 shows the evolution of the energy density of dark energy with redshift for one typical parameter in our models. Our results also satisfy the recent most stringent constraints which comes from the results of SN Ia, CMB and LSS [23].

model predictions.

IV. CONCLUSION

In conclusion, we have presented a Big Bang model of the Universe dominated by dilaton field which can mimic the matter (including dark matter) and dark energy. The model predicted age of the Universe, transition redshift, BBN and evolution of dark energy agree with current observations. Future higher quality data, and especially from SN Ia data and GRB data (because GRBs are produced predominantly in the early Universe [24]) at higher redshifts may constrain the cosmic evolution of matter, dark energy and Hubble constant more precisely and make critical tests on our

[1] A. G. Riess et al., Astron. J., **116**, 1009 (1998)

[2] S. Perlmutter et al., ApJ, **517**, 565 (1999)

[3] P. de Bernardis et al., Nature, **404**, 955 (2000)

[4] S. Hanany et al., ApJ, **545**, L5 (2000)

[5] D. N. Spergel et al. (WMAP), ApJS, **148**, 175 (2003)

[6] M. Tegmark et al., Phys. Rev. **D69**, 103501 (2004)

[7] L. M. Krauss and M. S. Turner, Gen. Rel. Grav. **27**, 1137 (1995); J. P. Ostriker and P. J. Steinhardt, Nature **377** 600 (1995); A. R. Liddle, D. H. Lyth, P. T. Viana and M. White, Mon. Not. Roy. Astron. Soc. **282** 281 (1996)

[8] Piazza and S.Tsujikawa, JCAP **0407**, 004 (2004); E.J.Copeland, M.Sami and S.Tsujikawa, hep-th/0603057; S. A. Bludman and M. Roos, Phys. Rev. **D65**, 043503 (2002); P. F. Gonzalez-Diaz,Phys. Lett. **B481**, 353 (2000); C. Rubano and P. Scudellaro, Gen. Rel. Grav. **34**, 307 (2002); S. Sen and T. R. Seshadri, Int. J. Mod. Phys. **D12**, 445 (2003); P. J. Steinhardt, Phil. Trans. Roy. Soc. Lond. A 361, **2497** (2003); L. A. Urena-Lopez and T. Matos, Phys. Rev. **D62**, 081302 (2000)

[9] C. Armendariz-Picon, V. Mukhanov, P. J. Steinhardt, Phys. Rev. **D63**, 103510 (2001); T. Chiba, Phys. Rev. **D66**, 063514 (2002); L. P. Chimento, A. Feinstein, Mod. Phys. Lett. **A19**, 761 (2004); M. Malquarti, E. J. Copeland, A. R. Liddle, M. Trodden M, Phys. Rev. **D67**, 123503 (2003); R. J. Scherrer, Phys.Rev.Lett. **93**, 011301 (2004)

[10] J. M. Aguirregabiria, R. Lazkoz, Phys.Rev. **D69**, 123502 (2004) ; J. S. Bagla, H. K. Jassal, T. Padmanabhan, Phys. Rev. **D67**, 063504 (2003); D. Choudhury, D. Ghoshal, D. P. Jatkar, S. Panda, Phys. Lett. **B544**, 231 (2002); A. V. Frolov, L. Kofman, A. A. Starobinsky, Phys. Lett. **B545**, 8 (2002); G. W. Gibbons, Class. Quant. Grav. **20**, S321 (2003); V. Gorini, A. Y. Kamenshchik, U. Moschella, V. Pasquier, Phys.Rev. **D69**, 123512 (2004); C. J. Kim, H. B. Kim, Y. B., Phys. Lett. **B552**, 111 (2003) ; T. Padmanabhan, Phys. Rev. **D66**, 021301 (2002); A. Sen, hep-th/0312153; G. Shiu, I. Wasserman, Phys. Lett. **B541**, 6 (2002)

[11] B. Boisseau, G. Esposito-Farese, D. Polarski, Alexei A. Starobinsky, Phys. Rev. Lett. 85, 2236 (2000); R. R. Caldwell, Phys. Lett. **B545**, 23 (2002); S. M. Carroll, M.

Hoffman, M. Trodden, Phys. Rev. **D68**, 023509 (2003); J. M. Cline, S. Y. Jeon, G. D. Moore, Phys.Rev. **D70**, 043543 (2004); P. DabrowskiM, T. Stachowiak, M. Szydlowski, Phys. Rev. **D68**, 103519 (2003); Z. Y. Sun and Y. G. Shen, Gen. Rel. Grav. **37**, 243 (2005); P. F. Gonzalez-Diaz , Phys. Rev. **D68**, 021303 (2003); Z. K. Guo et al, Phys. Lett. **B608**, 177 (2005); J. G. Hao, X. Z. Li, Phys. Rev. **D68**, 043501 (2003) ; S. Nojiri, S. D. Odintsov, Phys. Lett. **B562**, 147 (2003); R. G. Cai and A. Z. Wang, JCAP **0503**, 002 (2005);P. Singh, M. Sami, N. Dadhich, Phys. Rev. **D68**, 023522 (2003)

[12] C. P. Burgess, Int. J. Mod. Phys. **D12**, 1737 (2003); K. A. Milton, Grav. Cosmol. **9**, 66 (2003); K. Uzawa, J. Soda, Mod. Phys. Lett. **A16**, 1089 (2001)

[13] E. Elizalde, S. Nojiri, S.D. Odintsov, Physical Review **D70**, 043539 (2004); E. Elizalde, S. Nojiri, S.D. Odintsov, and P. Wang, Physical Review **D71**, 103504 (2005); V. F. Cardone, A. Troisi, S. Capozziello, Phys. Rev. **D69**, 083517 (2004); P. F. Gonzalez-Diaz, Phys. Lett. **B562**, 1 (2003); T. Padmanabhan, T. R. Choudhury, Phys. Rev. **D66**, 081301 (2002); M. Susperregi, Phys. Rev. **D68**, 123509 (2003)

[14] C. J. Gao and S. N. Zhang, Phys. Lett. **B605**, 185 (2004); C. J. Gao and S. N. Zhang, Phys. Rev. **D70**, 124019 (2004)

[15] M. Tegmark, M. A. Strauss and M. R. Blanton et al, Phys. Rev. **D69**, 103501 (2004)

[16] A. G. Reiss, L. G. Strolger and J. Tonry et al, Astrophys. J. **607**, 665 (2004)

[17] L. M. Krauss, astro - ph/0301012, 2003

[18] R. Cayrel et al., Nature, **409**, 691 (2001)

[19] C. L. Gardner, Nucl. Phys. **B707**, 278 (2005)

[20] C. Firmani, G. Ghisellini, G. Ghirlanda), V. Avila-Reese, astro-ph/0501395, MNRAS (in press)

[21] U. Alam, V. Sahni and A. A. Starobinsky, JCAP **0406**, 008 (2004); Z. H. Zhu, M. K. Fujimoto and X. T. He, Astrophys. J. **603**, 365 (2004);

[22] E. W. Kolb and M. S. Turner, The Early Universe, (Redwood City: Addison-Wesley Publishing Company, 1990)

[23] H. K. Jassal, J. S. Bagla, T. Padmanabhan, Mon. Not. Roy. Astron. Soc. **356**, L11 (2005); Y. Wang and M. Tegmark, Phys. Rev. Lett. **92**, 241302 (2004); Y.

Wang and K. Freese, astro-ph/0402208; Y. Wang and M. Tegmark, astro-ph/0501351; D. Huterer, A. Cooray, Phys. Rev. **D71**, 023506 (2005); A. Upadhye, M. Ishak, P. J. Steinhardt, astro-ph/0411803.

[24] J. R. Lin, S. N. Zhang and T. P. Li, Astrophys. J. **605**, 812 (2004); Z. G. Dai, E. W. Wang and D. Xu, Astrophys. J. **612**, L101 (2004).